



DEPARTMENT OF EDUCATION

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Superintendent of Public Instruction

Working Together for Student Success



Indiana Academic Standards Mathematics: PreCalculus: Trigonometry



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Introduction

The Indiana Academic Standards for Mathematics are the result of a process designed to identify, evaluate, synthesize, and create the most high-quality, rigorous standards for Indiana students. The standards are designed to ensure that all Indiana students, upon graduation, are prepared for both college and career opportunities. In alignment with Indiana's Every Student Succeeds Act (ESSA) plan, the academic standards reflect the core belief that all students can achieve at a high level.

What are the Indiana Academic Standards?

The Indiana Academic Standards are designed to help educators, parents, students, and community members understand what students need to know and be able to do at each grade level, and within each content strand, in order to exit high school college and career ready. The academic standards should form the basis for strong Tier 1 instruction at each grade level and for each content area for all students, in alignment with Indiana's vision for Multi-Tiered Systems of Supports (MTSS). While the standards have identified the academic content or skills that Indiana students need to be prepared for both college and career, they are not an exhaustive list. Students require a wide range of physical, social, and emotional support to be successful. This leads to a second core belief outlined in Indiana's ESSA plan that learning requires an emphasis on the whole child.

While the standards may be used as the basis for curriculum, the Indiana Academic Standards are not a curriculum. Curricular tools, including textbooks, are selected by the district/school and adopted through the local school board. However, a strong standards-based approach to instruction is encouraged, as most curricula will not align perfectly with the Indiana Academic Standards. Additionally, attention should be given at the district and school-level to the instructional sequence of the standards as well as to the length of time needed to teach each standard. Every standard has a unique place in the continuum of learning - omitting one will certainly create gaps - but each standard will not require the same amount of time and attention. A deep understanding of the vertical articulation of the standards will enable educators to make the best instructional decisions. The Indiana Academic Standards must also be complemented by robust, evidence-based instructional practices, geared to the development of the whole child. By utilizing well-chosen instructional practices, social-emotional competencies and employability skills can be developed in conjunction with the content standards.

Acknowledgments

The Indiana Academic Standards have been developed through the time, dedication, and expertise of Indiana's K-12 teachers, higher education professors, and other representatives. The Indiana Department of Education (IDOE) acknowledges the committee members who dedicated many hours to the review and evaluation of these standards designed to prepare Indiana students for college and careers.

PROCESS STANDARDS FOR MATHEMATICS

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.

PROCESS STANDARDS FOR MATHEMATICS	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason

	inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.
PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They

	express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern or structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

MATHEMATICS: Precalculus: Trigonometry

Unit Circle	
TR.UC.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
TR.UC.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
TR.UC.3	Use special triangles to determine the values of sine, cosine, and tangent for $\pi/3$, $\pi/4$, and $\pi/6$. Apply special right triangles to the unit circle and use them to express the values of sine, cosine, and tangent for x , $\pi \pm x$, and $2\pi \pm x$ in terms of their values for x , where x is any real number.

Triangles

TR.T.1	Define and use the trigonometric ratios (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles and the coordinates on the unit circle.
TR.T.2	Solve real-world problems with and without technology that can be modeled using right triangles, including problems that can be modeled using trigonometric ratios. Interpret the solutions and determine whether the solutions are reasonable.
TR.T.3	Explain and use the relationship between the sine and cosine of complementary angles
TR.T.4	Prove the Laws of Sines and Cosines.
TR.T.5	Understand and apply the Laws of Sines and Cosines to solve real-world and other mathematical problems involving right and non-right triangles.
TR.T.6	Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line. Use the formula to find areas of triangles.

Periodic Functions	
TR.PF.1	Graph trigonometric functions with and without technology. Use the graphs to model and analyze periodic phenomena, stating amplitude, period, frequency, phase shift, and midline (vertical shift).
TR.PF.2	Model a data set with periodicity using a sinusoidal function and explain the parameters of the model.
TR.PF.3	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
TR.PF.4	Construct the inverse trigonometric functions of sine, cosine, and tangent by restricting the domain.
TR.PF.5	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Identities	
TR.ID.1	Prove the Pythagorean identity $\sin^2(x) + \cos^2(x) = 1$ and use it to find trigonometric ratios, given $\sin(x)$, $\cos(x)$, or $\tan(x)$, and the quadrant of the angle.
TR.ID.2	Verify trigonometric identities and simplify expressions using trigonometric identities.
TR.ID.3	Prove the addition and subtraction identities for sine, cosine, and tangent. Use the identities to solve problems.
TR.ID.4	Prove the double- and half-angle identities for sine, cosine, and tangent. Use the identities to solve problems.

Polar Coordinates and Complex Numbers

TR.PC.1	Understand and use complex numbers, including real and imaginary numbers, on the complex plane in rectangular and polar form, and explain why the rectangular and polar forms of a given complex number represent the same number.
TR.PC.2	State, prove, and use DeMoivre's Theorem.
TR.PC.3	Define polar coordinates and relate polar coordinates to Cartesian coordinates.
TR.PC.4	Translate equations from rectangular coordinates to polar coordinates and from polar coordinates to rectangular coordinates. Graph equations in the polar coordinate plane.

Vectors

Vectors	
TR.V.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $ \mathbf{v} $, $\ \mathbf{v}\ $).
TR.V.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
TR.V.3	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
TR.V.4	Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
TR.V.5	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\mathbf{v}_x, \mathbf{v}_y) = (c\mathbf{v}_x, c\mathbf{v}_y)$.
TR.V.6	Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\ c\mathbf{v}\ = c \mathbf{v} $. Compute the direction of $c\mathbf{v}$ knowing that when $ c \mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$).
TR.V.7	Solve problems involving velocity and other quantities that can be represented by vectors.